AN ENGINEERING SELECTION FOR SIZE OF LONG CONDUCTORS CONSIDERING FACTORS IN ADDITION TO NEC

Frontiers in Power, OSU, October 2000

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Abstract: Long power cables and large machines require different application techniques from most electrical wiring. Selection of conductor size must consider insulation temperature as well as voltage drop. The NEC is commonly used for determining conductor size. It is primarily based on current carrying capacity, since this impacts the temperature of the insulation.

However, operational considerations demand the consideration of other factors. The procedure is primarily a mathematical design practice that is seldom considered in manual calculations.

We have applied numerous considerations to develop a simple relationship that can be readily applied in routine designing. It incorporates wire diameter, length, current, number of phases, and temperature correction, with permissible voltage drop.

Background

Selection of conductor size has been a common part of electrical systems since their inception. The selection is generally based on tables from the *National Electrical Code* Article 310 [15]. Insulation temperature drives the NEC criteria. This is primarily dependent on current, which creates heat in the form of I^2R losses.

Typically the tables are adequate for runs less than 100 feet, but can yield an undersized wire for much longer distances. A critical component not included is voltage drop on long runs.

Conductor properties drive the additional criteria. This is primarily dependent on voltage.

Because power cables are a crucial part of most facilities, relationships have been developed to handle various combinations of factors that affect the voltage drop. The relationships address conductor size based on conductor resistivity, operating temperature, path length, current, and number of phases [2].

If the allowable voltage drop is known and the current requirement has been specified, then the permissible wire impedance can be calculated. Remember, the voltage drop in a branch circuit should be less than 3% and the total voltage drop in the feeder and branch should be less than 5%.

Because wire is so small the diameter is calculated in mils. One (1) mil is 0.001 inch. The area of the wire is circular mils. The relationship between diameter, mil^2 and circular mils is expressed below.

$$A(mil^2) = \frac{\pi d^2}{4} = 0.7854\, d^2$$

$$A(cmil) = d^2$$

$$mil^2 = 0.7854\, cmil$$

A wire 10 mils in diameter has an area of 100 cmils or 78.54 $mils^2$.

Resistance

The resistance '*R*' of a conductor is determined by the resistivity 'ρ' of the material, the cross-sectional area of the wire and the length of the run.

$$R = \rho \frac{\ell}{A}$$

$$\rho = R \frac{A}{\ell}$$

The resistivity depends upon the physical properties of the material, the temperature '*T*' of the conductor, and the configuration of the cable run. The reference temperature for material properties is 20°C. At that temperature, annealed copper wire has a resistivity of 10.371 Ω·cmil/ft [19]. For standard conduit or cable tray configurations, a configuration factor of 1.02 can be assumed. Thus, resistivity of copper can be determined by the relationship.

$$\rho = 10.371 * 102 \left(\frac{234.5^{o}C + T}{254.5^{o}C} \right)$$

For a typical installation at 25°C, a copper cable has a resistivity of 10.786 Ω·cmil/ft. Aluminum has a value of approximately 17.35 Ω·cmil/ft. [19]

Wire Dimensions, Resistance

The first pass estimate of wire size can be found by using the resistance value and Ohms law.

$$A = \frac{\rho\, \ell\, I}{V_D}$$

where 'V_D' represents the desired maximum voltage drop.

The length value ' ℓ ' includes the length of the conductor going to the load and returning. The effective length of the wire is the distance times the number of conductors per phase. The current is corrected by a phase factor for single-phase or three-phase.

For 1 phase: phase factor = 1 # conductors/phase = 2
For 3 phase: phase factor = √3 # conductors/phase = 1

An expanded relationship for wire area combines these factors where '*D*' represents the one-way distance of the wire run.

$$A = \frac{\rho\,(D * \#cond)(phase\,factor * I)}{V_D} \tag{1}$$

Manipulation of the wire dimensions equation can provide other design tools. If the wire size is known, the voltage drop in a wire can be found.

$$V_D = \frac{\rho\,(D * \#cond)(phase\,factor * I)}{A} \tag{2}$$

The distance a given size wire will carry current can be found by making another transposition.

$$D = \frac{V_D\,A}{\rho\,(\,\#cond)(phase\,factor * I)} \tag{3}$$

The resistivity calculated resistance is a dc value which has to be corrected for ac conditions. For sizes smaller than #4/0, the dc and ac values are about equal. For much larger sizes the Z_{ac} may be as much as 1.3 * R_{dc}.

AC Consideration

The total ac opposition involves inductive and capacitive reactance '*X*' which are combined to calculate impedance.

$$Z = R + j\,X$$

For impedance calculations, the inductive reactance is calculated. At 60 Hz frequency, all the coefficients can be combined.

$$X_L = 2\pi\,60\,L = 377\,L$$

The inductance is dependent on the permeability 'μ', area, and length.

$$L = \frac{\mu\,\ell}{A}$$

However, the inductance of two long, cylindrical conductors, parallel and external to each other should be used [4, 5, 6]. Calculate the constant from $\mu_0 / 2\pi = 4*10^{-7}$.

$$L = 4 * 10^{-7}\,\ell\left(1 + 4\ln\left(\frac{d}{r}\right)\right)$$

The distance' *d* ' is between the centers of the two conductors and '*r*' is the radius of the conductor cross section. Length is measured in feet, '*d*' and '*r*' are measured in inches.

For power line frequencies, the skin effect is the negligible so the ' 1' term drops out.

$$L = 4 * 10^{-7}\,\ell\,\ln\left(\frac{d}{r}\right)$$

For coax and transmission lines where the lines rotate, the average value is one-half the equation. This is appropriate for two, straight round conductors, or for per phase of a symmetric three-phase cable.

$$L = 2 * 10^{-7}\,\ell\,\ln\left(\frac{d}{r}\right)$$

For length in feet, '*d*' in inches and '*r*' in mils, the formula becomes

$$L = 13.816 * 10^{-7}\,\ell\,\ln\left(\frac{d}{r}\right)$$

The distance '*d* ' between the centers of the conductors can be estimated for individual cables in a cable tray or conduit.

*d = 2*insulation thickness + 2*r*

The capacitance reactance is calculated at 60 Hz.

$$X_C = \frac{1}{2\pi f\,C} = \frac{1}{377C}$$

Capacitance is determined from the permittivity 'ε' of the conductor material.

$$\frac{1}{C} = \frac{\ell}{\varepsilon A}$$

When parallel round conductors are used, the capacitance takes on a slightly different form [6]. This is adequate for the phase to neutral capacitance of a two-wire line or symmetrical three-phase cable. Between the two-wires of a single-phase line, the value would be one-half the equation.

$$C = \frac{2\pi\,\varepsilon\,\ell}{\ln(d\,/\,r)}$$

Capacitive reactance is negligible at voltages less than 2400 volts. Furthermore, the shunt impedance is very large for lines less than 10 miles, therefore, it is usually ignored for power cable calculations.

Voltage Drop, Impedance

Ohm's Law indicates that impedance is the ratio of volts '*V*' to amps '*I*'.

$$Z = \frac{V}{I}$$

The voltage drop is related to the impedance.

$$\frac{V_D}{I} = Z = R + j\,X$$

$$\left(\frac{V_D}{I}\right)^2 = R^2 + (X_L - X_C)^2$$

The underlying relationships use the distributed values.

$$\left(\frac{V_D}{I}\right)^2 = \left(\frac{\rho\ell}{4r^2}\right)^2 + \left(\frac{2\pi f\mu\,\ell}{A} - \frac{\ell}{2\pi f\,\varepsilon A}\right)^2$$

However, using the relationships for parallel conductors and neglecting capacitance, the following formula is derived.

$$\left(\frac{V_D}{I}\right)^2 = \left(\frac{\rho \ell}{4r^2}\right)^2 + \left(2\pi f \ell * 13.816 * 10^{-7} \ln\left(\frac{d}{r}\right)\right)^2$$

Wire Dimensions, Impedance

The voltage drop calculation is straight forward when the current and wire configuration is known.

$$V_D = I\sqrt{\left(\frac{\rho \ell}{4r^2}\right)^2 + \left(2\pi f \ell * 13.816 * 10^{-7} \ln\left(\frac{d}{r}\right)\right)^2} \qquad (4)$$

If the wire size and maximum voltage drop are known, the maximum length run of the circuit can be calculated.

$$\ell = \frac{V_D}{\sqrt{\left(\frac{\rho}{4r^2}\right)^2 + \left(2\pi f * 13.816 * 10^{-7} \ln\left(\frac{d}{r}\right)\right)^2}} \qquad (5)$$

Because the relationship involves both '*r*' and 'ln (r)', it cannot readily be solved directly for '*r*'. However, assumptions can be made which allow for a good estimation.

For industrial cables, the thickness of the insulation is approximately one-half the radius of the conductor [1,5]. Therefore, for conductors in conduit or cable tray, the distance '*d* ' is approximately 3r. This is true for conductors #4 AWG and larger. For smaller conductors, '*d* ' is approximately 5r. Assuming power cable of at least #4 AWG, V_D then becomes

$$V_D = I\sqrt{\left(\frac{\rho \ell}{A}\right)^2 + \left(2\pi f \ell * 2 * 10^{-7} \ln\left(\frac{3r}{r}\right)\right)^2}$$

Given the desired maximum voltage drop, and the length of the run, the wire size can be determined from the following equation.

$$A = \sqrt{\frac{\rho^2}{\left(\frac{V_D}{I \ell}\right)^2 - \left(2\pi f * 2 * 10^{-7} \ln(3)\right)^2}} \qquad (6)$$

Comparison of Procedures

This will provide adequate design for a single machine or load. Two methods should be used to select wire for installations longer than 100 feet. First, incorporate the NEC considerations of insulation temperature based on current. Next calculate this method of conductor size based on voltage drop. Select a wire size that is the larger of this voltage drop calculation or the current calculation from the NEC.

Summary

The technique developed is determination of wire size dependent on conductor properties. The first technique considers both resistance and inductance. The resistivity of the wire is corrected for material, configuration, and temperature. The area of the wire is calculated from resistivity, length of current path, number of phases, current, and voltage drop. The equation is also manipulated to yield voltage drop or distance based on the conductor parameters.

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